Motivation
The quantitative predictions of multiphase flow dynamics with more than two fluids is of practical interest in many subsurface processes. This covers various fields like petroleum engineering, simulation of contaminant intrusions, or geological CO2 sequestration. In this project, three-phase flow (e.g., oil, water, and gas) in porous media is considered. Furthermore, an efficient numerical method based on the heterogeneous multiscale approach will be developed.

Physical and Mathematical Model
By neglecting gravitational forces and assuming the fluids to be immiscible and incompressible, the governing equations are given by mass balance laws. In the capillarity-free case, these laws lead, in the fractional flow formulation, to a first-order system of nonlinear evolution equations with the unknown phase saturations and a coupled Darcy-type pressure—velocity system. This model is called the macroscale model.

Unfortunately, this system can have multiple weak solutions: there is a whole family of solutions involving non-classical transitional waves (LeFloch, 2002). These shock waves are indeed not mathematical artefacts, but have been experimentally observed as leading fronts and overshoots caused by injection (DiCarlo, 2004). In this project, we are interested in the unique, physically relevant solution which is selected as the sharp-interface limit of solutions of regularised systems.

For the regularised system, capillary pressure effects have to be added, which are not only modelled by the equilibrium pressure alone, but should also involve a rate-dependent contribution (Hassanizadeh, 1993). The regularised model is called microscopic model.

Objectives
The objectives can be summarised in an analytical and a numerical part.

(a) Study of the sharp-interface limit for rate-dependent capillary pressure: On the theoretical side, the sharp-interface limit for rate-dependent capillary pressure in the case of three phases will be studied. The rate-dependent models for two-phase flow have been successfully analysed, but there are almost no analytical and numerical results available for three phases. For the analysis, not only classical tools will be applied but also more recently developed methods in the field of hyperbolic conservation laws, e.g., compensated compactness and kinetic decomposition will be adopted.

(b) Development, validation, and application of a heterogeneous multiscale method (HMM): The goal is to develop and implement a heterogeneous multiscale method covering transitional waves in multiple space dimensions to solve the system mentioned above. A heterogeneous multiscale method is necessary because classical numerical methods for the capillarity-free model which rely on a Riemann-solver based finite-volume scheme or a discontinuous Galerkin idea will not work: the selection of the approximated weak solution is determined from the inherent numerical dissipation and not from the augmented model with dynamic capillarity effects. However, it is too
extensive to solve the regularised system over the whole domain with a fine mesh, and, above all, the solution of the regularised systems is only relevant close to a transitional wave. It is thus adequate to use the non-regularised systems elsewhere. This method will be tested on infiltration flow problems and especially to simulate a water-alternating gas (WAG) injection.

**Current state of the work**

**(a)** The above-mentioned method of kinetic decomposition has already been used successfully for a non-local regularisation of nonlinear hyperbolic conservation laws in several space variables (Kissling, 2009).

**(b)** A new mass-conserving numerical method based on a heterogeneous multiscale approach for scalar model problems in one dimension has been developed and tested which can drive nonclassical shock waves, i.e., overshoots. Furthermore, the enormous time difference between solving the whole regularised equation and solving the regularised model only next to the nonclassical shock has been shown. The new algorithm has been tested for applications from phase transition theory as well as from two-phase flow in porous media (Kissling, 2009).

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<th>time T</th>
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<td>7.98 sec.</td>
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*CPU-Times for the HMM versus CPU-times, which would be required using the microscale model for the whole computational domain*

Numerical solution of the Buckley-Leverett equation with the same initial condition for different numerical schemes. Left: Capillarity-free model with a classical finite volume scheme. Right: Heterogeneous multiscale approach. One can clearly see that the left one cannot drive nonclassical waves, i.e., no saturation overshoot occurs in contrast to the right one.

**References**