Description of the master's thesis

The aim of my thesis was to reproduce the spatial dependence structures of hydraulic conductivity fields based on theoretical copula models. The hydraulic conductivity data set from two sandy aquifers in Canada were considered, Borden and North Bay, each comprising ca. 1200 measurements in orthogonal cross sections.

A maximum Gauss copula was analytically developed, which offers more flexibility and additional structures compared to other more traditional Gaussian copulas. For example, a normal bivariate Gaussian copula density function is symmetric, which means that for a pair of very small values and a pair of very high values the copula density values are bound to be identical. However, reality often does not exhibit this characteristic feature of symmetry. Therefore, theoretical maximum Gauss copula density functions, due to their asymmetric characters, are likely to better reproduce empirical copula density functions. In order to understand how to build a maximum Gaussian density function and why it includes the attribute “maximum”, it is helpful to think of two spatial fields of data points. For each coordinate, the two data points of the two fields were compared and the higher (maximum) value at this coordinate was selected. If this is done for every coordinate of the field, a new maximum field has been generated based on the two initial fields. The procedure of constructing a maximum Gauss copula density function analytically can be separated in two parts. At first, the calculation of the maximum Gauss density must be conducted based on two bivariate normal density fields. In a second step, the final maximum Gauss copula was calculated by using the calculated maximum Gauss density functions and its marginal densities.
After the theoretical function of maximum Gauss copula has been determined, the parameters of the theoretical copula needed to be fitted to the hydraulic conductivity fields of the real world aquifers, Borden and North Bay. These parameters were then incorporated into a geostatistical model and simulation program, gstat [1], to generate Gauss field of hydraulic conductivity. In order to generate fields which are highly similar in their spatial structure to the real world aquifers, the optimization of the parameters of the theoretical maximum Gauss copula density function was crucial. The aim of the optimization process was to find a reliable and stable solution, which results in the best fitting theoretical copula density function to the empirical copula density function.

In a next step, the theoretical spatial fields of hydraulic conductivity derived from the maximum Gauss copula were compared with features of observed spatial fields of hydraulic conductivity of real world aquifers. The generated maximum Gauss fields based on transformation parameters were able to reproduce anisotropic structures which were also found in the hydraulic conductivity fields in the Borden aquifer in reality. However, in some of the maximum Gauss fields the anisotropic structure was too extreme. In order to measure to which extent the simulated hydraulic conductivity fields agreed with the observed measurement, the empirical rank correlation functions were compared with the rank correlation function based on the simulated fields. It was found for both aquifers, Borden and North Bay, that the generated maximum Gauss fields showed high agreement with the observed aquifer in terms of their rank correlation functions. Another evaluation measure was applied by calculating the spatial moments of the hydraulic conductivity fields. The spatial moments described the shape and transport properties of processes, for which advection as well as diffusion and/or dispersion play an important role, as described in [2]. HydroGeoSphere [3], a three-dimensional numerical model for subsurface flow and solute transport was applied and the simulated maximum Gauss fields of hydraulic conductivity were uploaded into the model. Subsequently, numerical tracer tests were conducted. The zeroth spatial moment represented the total mass of tracer in the domain. The first spatial moment was a measure for the transport velocity and the second spatial moment represented the dispersion coefficient. It was found for the first spatial moment, that the velocity values were higher for the simulated fields based on the Borden aquifer compared to the simulated fields derived from the North Bay aquifer. This can be explained by the difference in the observed hydraulic conductivity values between the two aquifers. Regarding the second spatial moment, which represented the Dispersion coefficient, the values not only differed between the two different aquifers, Borden and North Bay, but also between simulated hydraulic conductivity based on the same aquifer depending on the transformation parameters.
References

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Flow and Solute Transport in Discretely-Fractured Porous Media. Journal of Contaminant