Description of the master's thesis

The dynamics of two immiscible incompressible fluids in a porous medium can be described by the fractional flow equations for the wetting fluid saturation, the total velocity, and a global pressure. If the effects of capillary pressure are neglected, this gives a mixed-type system that consists of a nonlinear hyperbolic conservation law coupled to a nonlinear elliptic equation for the pressure. In particular the numerical simulation of the elliptic part can be extremely expensive. For this reason much effort has been put into deriving more simple models that approximate special flow regimes.

In this thesis we are interested in almost parallel flow regimes, also called vertical flow equilibria. Based on work by Yortsos [3], the given system is non-dimensionalized and the ratio of vertical to horizontal length is identified as a characteristic number. By means of formal asymptotic analysis a limit model can be determined that consists of only one first-order transport equation for the saturation. In other words, the elliptic equation for the pressure is eliminated. However, the nonlinear transport equation contains a nonlocal integral term in the nonlinear flux.

The first result of the thesis is a correction of the scaling proposed by Yortsos which changes the asymptotic analysis but not the form of the final equation. The second major new contribution is the design and implementation of a finite-volume scheme for the non-local conservation law on structured meshes. The crucial step is the formulation of CFL-like conditions for the time step that allows stable computations. We have been able to show that the method is stable for the maximum norm and a BV seminorm. In turn, these results could be taken to prove the convergence of the finite-volume scheme. In fact this has been done following the general concept of BV-stability (see [2]) afterwards in [1].

In the spirit of the original idea that the special flow regime should be governed by a more easily accessible mathematical model we have concluded the work with various numerical experiments in two spatial dimensions. The basic set-up is an infiltration from the left vertical boundary. Typical fingering behaviour could be detected. We have shown that the limit model enables computational times that are magnitudes smaller than for the full mixed-type fractional flow model. It could be anticipated that the nonlocal operator in the flux regularizes the solutions. Our numerical experiments suggest that this is not the case: we can observe typical shock-wave and rarefaction wave solution patterns.
References

